Effective Lagrangian in de Sitter Spacetime

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Introduction

Cosmological constant (Dark energy) problem:

$$H^2 M_P^2 \sim \Lambda^4$$

$$\Lambda \sim m_\nu$$

$$(\frac{\Lambda}{M_P})^4 = (\frac{H}{M_P})^2 \sim 10^{-120}$$

There still may be large unknown territory in nonperturbative and nonequilibrium physics.

Scale invariant fluctuation

$$\phi_{\mathbf{p}}(x) = \frac{H\tau}{\sqrt{2p}}\left(1 - i\frac{1}{p\tau}\right)e^{-ip\tau+i\mathbf{p}\cdot\mathbf{x}}.$$

In de Sitter space, scale independent metric fluctuations are universally generated.

$$< \delta g \delta g >= \frac{H^2}{M_P^2}\int_{1/a}^{H} dP/P \sim \frac{H^2}{M_P^2}\log a(t) \sim \frac{H^2}{M_P^2}Ht$$

We assume the initial scale of the universe to be O(H)

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad a(t) = e^{Ht},$$

$$(g_{\mu\nu})_{\mathrm{dS}} = a^2(\tau)\eta_{\mu\nu}, \quad a(\tau) = -\frac{1}{H\tau}.$$

$$g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu}, \qquad \Omega(x) = a(\tau)e^{\kappa w(x)},$$

$$\det \tilde{g}_{\mu\nu} = -1, \qquad \tilde{g}_{\mu\nu} = (e^{\kappa h(x)})_{\mu\nu},$$

We use the following metric propagator in a particular gauge

$$\langle X(x)X(x')\rangle = -\langle\varphi(x)\varphi(x')\rangle,$$

$$\langle\tilde{h}^i{}_j(x)\tilde{h}^k{}_l(x')\rangle = (\delta^{ik}\delta_{jl} + \delta^i{}_l\delta_j{}^k - \frac{2}{3}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle,$$

$$h^{00} \simeq 2w \simeq \frac{\sqrt{3}}{2}X.$$

$$< \varphi^2 >= \frac{H^2}{4\pi^2}\log a(\tau)$$

N. C. Tsamis and R. P. Woodard

We have investigated IR logarithmic effects
in Schwinger-Keldysh formalism where the both
metric and matter are quantized.
The matter fields do not give rise to such effects
unless it is a minimally coupled massless scalar field.
It thus appears to be enough to integrate metric
fluctuations only.

Kitamoto, Kitazawa 2013

We construct the effective Lagrangian as

$$\mathcal{L}_{eff} =< \mathcal{L} >_{metric}$$

where the average is taken over the metric only.

We consider generic renormalizable Lagrangian

$$\mathcal{L}(\lambda, g_Y, e) = \sqrt{-g}\big[-\frac{1}{2}g^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{2}m^2\phi^2 + i\bar{\psi}e^\mu{}_a\gamma^a D_\mu\psi + m_f\bar{\psi}\psi - \frac{1}{4!}\lambda_4\phi^4 - \lambda_Y\phi\bar{\psi}\psi - \frac{1}{4e^2}g^{\mu\rho}g^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\big].$$

We rescale

$$\phi \to a^{-1}e^{(\alpha-1)\omega}\phi$$
$$\psi \to a^{-\frac{3}{2}}e^{(\beta-\frac{3}{2})\omega}\psi$$

Gauge invariance keeps the gauge field intact

$$-\frac{1}{2}e^{2\alpha\omega}\tilde{g}^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{2}m^2a^2e^{2(1+\alpha)\omega}\phi^2 + ie^{2\beta\omega}\bar{\psi}\tilde{e}^\mu{}_a\gamma^a D_\mu\psi + m_f a e^{(1+2\beta)\omega}\bar{\psi}\psi - \frac{1}{4!}e^{4\alpha\omega}\lambda_4\phi^4 - \lambda_Y e^{(\alpha+2\beta)\omega}\phi\bar{\psi}\psi - \frac{1}{4e^2}\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

We adjust α and β to preserve Lorentz invariance at sub-horizon scale.

This requirement fixes α and β uniquely
We reproduces the identical results with Schwinger-Keldysh formalism obtained before.
It clearly shows that IR logarithmic effects are local and makes couplings time dependent.

In general relativity, Lorentz symmetry must hold at least locally when the spacetime curvature can be ignored.
In fact the requirement of Lorentz symmetry at this limit follows from the fundamental principle of general relativity.

$$-\frac{1}{2}\tilde{g}^{00}e^{2\alpha\omega}\partial_0\phi\partial_0\phi-\frac{1}{2}\tilde{g}^{ij}e^{2\alpha\omega}\partial_i\phi\partial_j\phi$$

The α=0 contribution is

$$-\frac{3}{16}<\varphi^2>\partial_0\phi\partial_0\phi-\frac{13}{16}<\varphi^2>\partial_i\phi\partial^i\phi$$

The linear term in α is

$$-\frac{3\alpha}{8}<\varphi^2>\partial_0\phi\partial_0\phi-\frac{\alpha}{8}<\varphi^2>\partial_i\phi\partial^i\phi$$

We find that the requirement of Lorentz invariance fixes α=-2.

The total result including α^2 effect is

$$-\frac{3}{16} < \varphi^2 > \partial_0\phi\partial_0\phi + \frac{3}{16} < \varphi^2 > \partial_i\phi\partial^i\phi$$

We thus find that the IR logarithmic effect can be cancelled by the time dependent wave function renormalization of

$$\phi \rightarrow Z\phi \text{ where } Z^2 = (1 + \tfrac{3}{8} < \varphi^2 >)$$

Let us consider the scalar quartic coupling in our parametrization with canonically normalized kinetic term.

$$\lambda\phi^4 \to \lambda Z^4 e^{-8\omega}\phi^4$$

$$\lambda Z^4 < e^{-8\omega} >= \lambda(1 - \frac{21}{4} < \varphi^2 >)$$

We find that the coupling decreases with time

We next consider the mass term

$$m^2 e^{2\omega}\phi^2 \to m^2 e^{-2\omega} Z^2 \phi^2$$

Since $< e^{-2\omega} > Z^2 \sim 1$, the mass term is not renormalized after the wave function renormalization.

In this way, we obtain

$$(\lambda_4)_{\text{eff}} \simeq \lambda_4\{1 - \frac{21\kappa^2 H^2}{16\pi^2}\log a(\tau)\},$$

$$(\lambda_Y)_{\text{eff}} \simeq \lambda_Y\{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\}.$$

$$e_{\text{eff}} \simeq e\{1 - \frac{3\kappa^2 H^2}{16\pi^2}\log a(\tau)\}.$$

They agree with the results obtained in the Schwinger Keldysh formalism where $\alpha=\beta=0$.

Effective Lagrangian approach in de Sitter spacetime leads to unique physical predictions by imposing Lorentz invariance at sub-horizon scale.

Matter fields are accompanied by soft metric fluctuations and we need to take such effects properly.
Such a requirement is intimately connected with unitarity and Lorentz invariance.

We need deeper understanding of gauge dependence

Although this is a small effect at present, it could lead to observable effects as couplings may be incredibly fine tuned.